**CHAPTER 5**

**Exercise Set 5.2**

**A math problem with numbers and equations

Description automatically generated**

1. P(1) is “12 =   
   P(1) is true because  
   12 = 1 and == 1 also.  
     
   b. P(k) is “12 + 22 +· · ·+k2 =   
     
   c. P(k + 1) is “12 + 22 +· · ·+(k+1)2=   
     
   d. Must show: If for some integer k ≥ 1,  
   12 + 22 +· · ·+k2 =   
   then  
   12 + 22+⋅⋅⋅+(k+1)2 =

**Exercise Set 5.3**

**A math problem with numbers and symbols

Description automatically generated**

Based on the existing pattern, we can write a general formula as:  
  
1/1⋅3+1/3⋅5+⋯+1 / ((2n−1)⋅(2n+1)) = n2n+1  
  
Proof: Let P(n) be the above statement.  
1. For n=1, we have LHS=1/1⋅3=1/3 and RHS=1/(2(1)+1)=1/3=LHS, thus it is true for P(1).  
2. Assume it is true for P(k), that is:  
1/1⋅3 + 1/3⋅5 +⋯+ 1/((2k−1)⋅(2k+1)) = k/(2k+1)  
3. For n=k+1, we have:  
LHS=1/1⋅3+1/3⋅5+⋯+1/((2k−1)⋅(2k+1))+1/((2k+1)⋅(2k+3)) = k/(2k+1) + 1/((2k+1)⋅(2k+3)) = (2k2+3k+1)/((2k+1)⋅(2k+3)) = ((2k+1)(k+1))/((2k+1)⋅(2k+3)) = (k+1) / (2k+3) = RHS  
4. Thus P(k+1) is also true and we proved the statement by mathematical induction.

**Exercise Set 5.6**

**A white background with black text

Description automatically generated**

1.

a2=2a2−1+2 = 2.a1+2 = 2.1+2 = 4  
a3=2a3−1+3 = 2.a2+3 = 2.4+3 = 11  
a4 =2a4−1 +4 = 2.a3+4 = 2.11+4 =26

2.

b1 = 1  
b2 = 1 + 3(2) = 7  
b3 = 7 + 3(3) = 16  
b4 = 16 + 3(4) = 28

3.

c0=1,c1=1⋅(c0)2=1⋅(1)2=1

c2=2(c1)2=2⋅(1)2=2

c3=3(c2)2=3⋅(2)2=12

**Exercise Set 5.7**

A white paper with black text

Description automatically generated

3.

4.

bk = bk-1/(1+bk-1), for all integers k ≥ 1, b0 = 1

b1 = b0/(1+b0) = 1/(1+1) = 1/2  
b2 = b1/(1+b1) = (1/2)/(1+(1/2)) = 1/2/(3/2) = 1/2 \* 2/3 = 1/3  
b3 = b2/(1+b2) = (1/3)/(1+(1/3)) = 1/3/(4/3) = 1/3 \* 3/4 = 1/4  
b4 = b3/(1+b3) = 1/4/(1+(1/4)) = 1/4/(5/4) = 1/4\*4/5 = 1/5  
bk = 1/(k+1)

5.

c₁=1  
c₂=3(1)=3  
c₃=3(3)=9  
c₄=3(3⋅3)=27  
  
cₙ=3ⁿ

**Exercise Set 5.7**

A math equations on a white background

Description automatically generated

8.

9.

10.